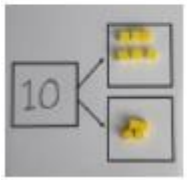
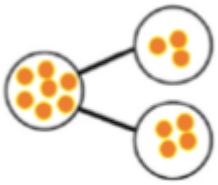
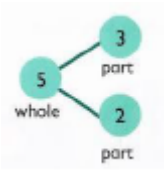
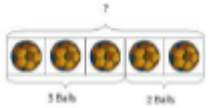
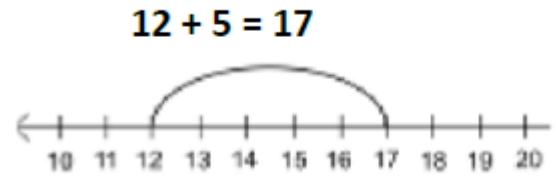
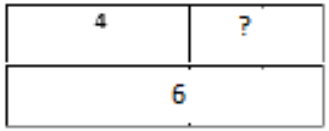
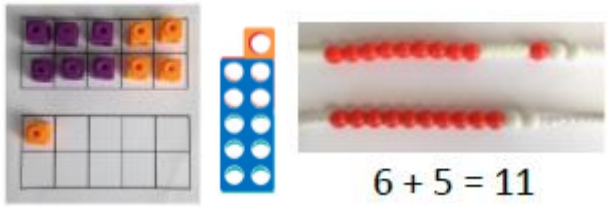
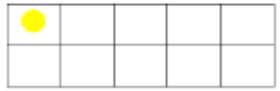
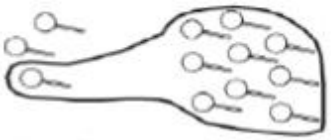
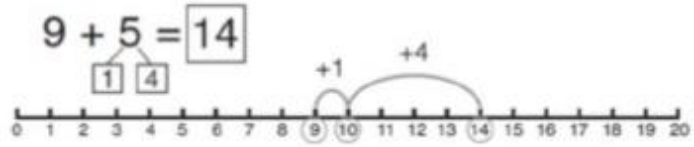
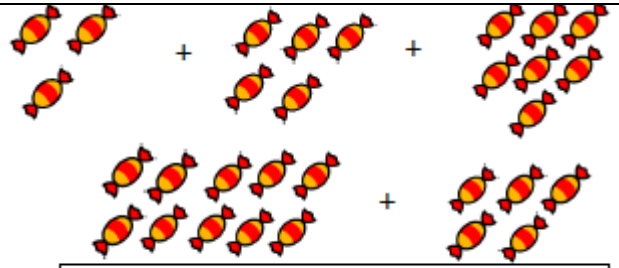
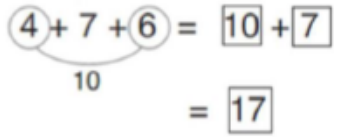
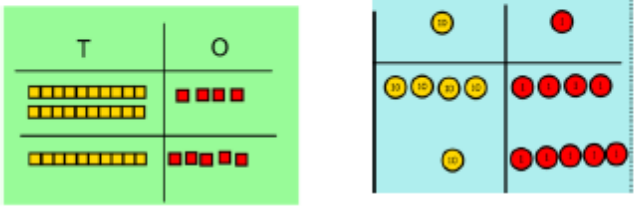
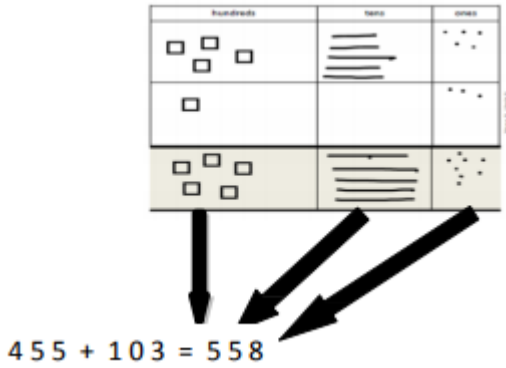
Addition

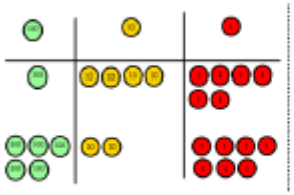
Objective	Concrete	Pictorial	Abstract
Combining two parts to make a whole	  Use cubes to add two numbers together as a group or in a bar.  	    	4 + 3 = 7 Four is a part, three is a part and the whole is seven. Addend + addend = sum 7 = 4 + 3
Counting on from the biggest number	  	 	12 + 5 = 17 17 = 12 + 5 4 + 2 = 6 6 = 4 + 2

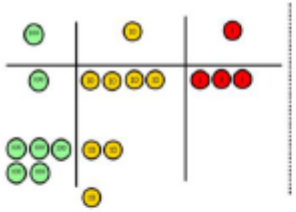
Regrouping to make 10	6 + 5  	   3 + 9 =  	9 + 5 = 14 9 + 1 + 4 = 14 7 + 4 = 11 7 + 3 + 1 = 11 If I am 7, how many more do I need to make 10? How many more do I need to add on now? Can I use my number bonds? Can I regroup?
Adding three single digits	**4 + 7 + 6= 17** Put 4 and 6 together to make 10. Add on 7. 	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	

<table>
<tr>
<td>Column addition - Partitioning to add without regrouping</td>
<td>
24 + 15=
Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.


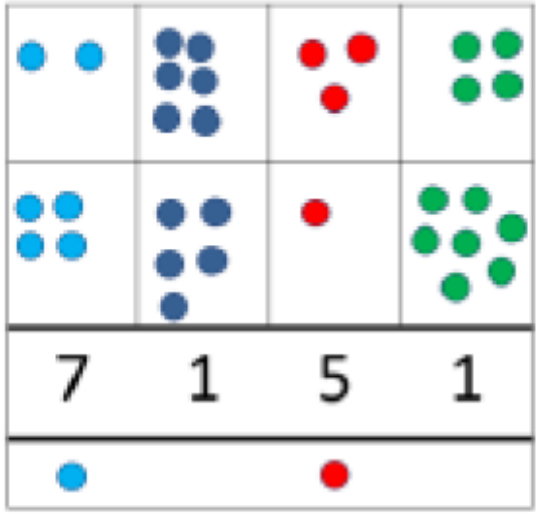
</td>
<td>


</td>
<td>
24 + 15 = ?
20 + 10 = 30
4 + 5 = 9
30 + 9 = 39

24
+ 15
39
</td>
</tr>
<tr>
<td>Column addition - with regrouping (Year 2+)</td>
<td>
Make both numbers on a place value grid.

146
+ 527

Add up the ones and exchange 10 ones for one 10.

146
+ 527

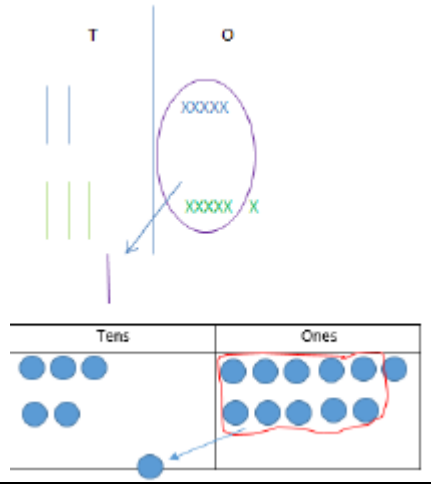
Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.
</td>
<td>
Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.


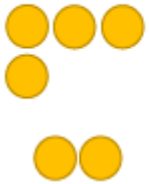
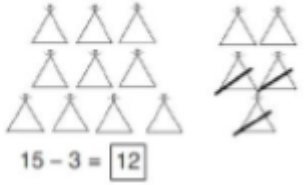
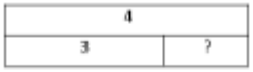
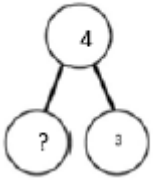
</td>
<td>
Looking for ways to make 10
36 + 25=
1 5
30 + 20 = 50
5 + 5 = 10
50 + 10 + 1 = 61

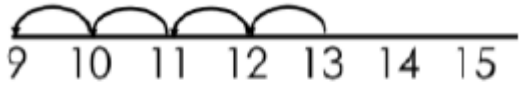
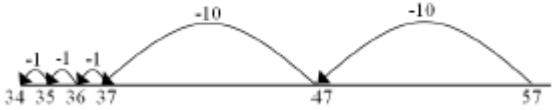
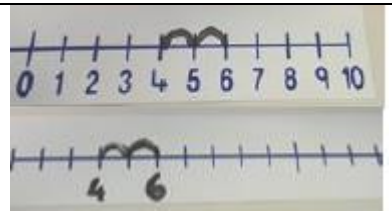
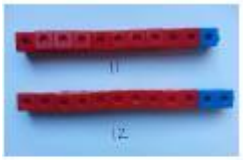
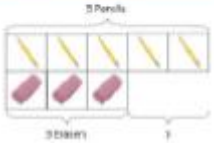
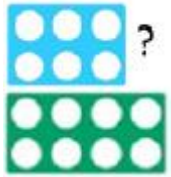
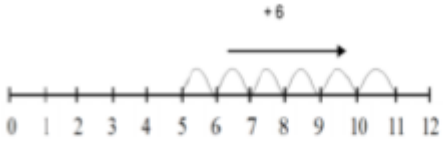
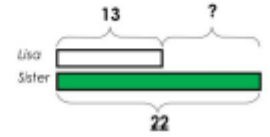
20 + 5
40 + 8
60 + 13 = 73

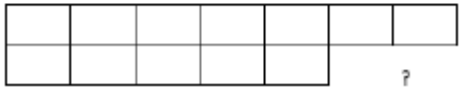
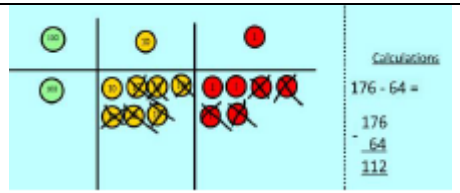
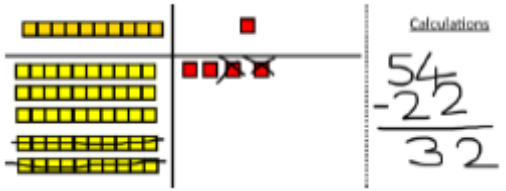
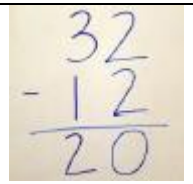
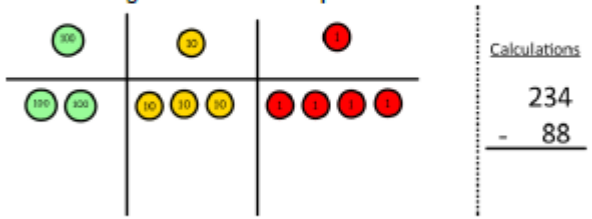
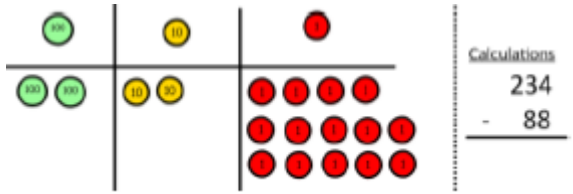
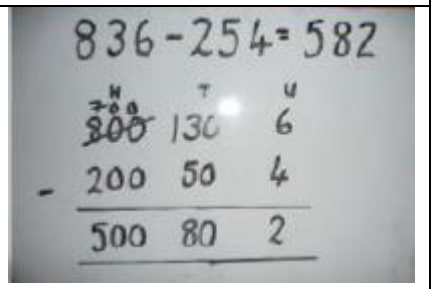
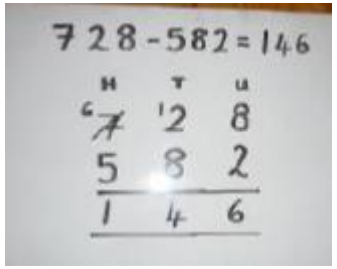
536
+ 85
621
1 1
</td>
</tr>
</table>

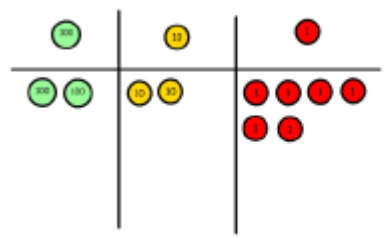
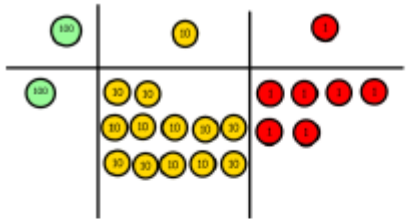
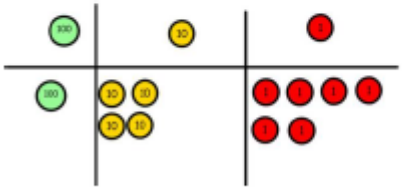
		 T O XXXXX XXXXX X Tens Ones	

Subtraction

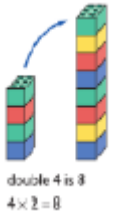
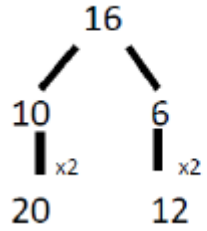
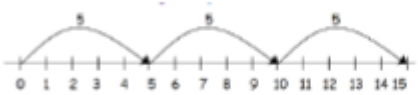
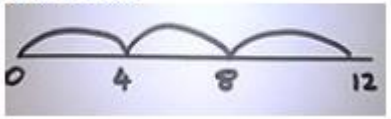
Objective	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4	Cross out drawn objects to show what has been taken away.   15 – 3 = 12 Children to draw the concrete resources they are using and cross out.  Use of the bar model:   	4 - 3 = □ = 4 - 3

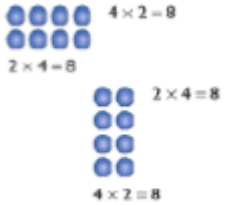
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. **13 – 4** 	 Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	 Six subtract four is two. Minuend – subtrahend = difference
Find the difference	  	 Count on to find the difference. Draw bars to find the difference between 2 numbers. **Comparison Bar Models** *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 	The difference between 8 and 6 is 2. Children to also explore why 9 – 7 = 8 – 6 (the difference of each digit has changed by 1).

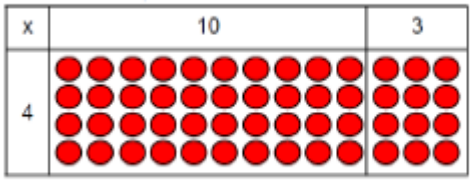
		Children to draw the cubes/other concrete objects which they have used XXXXXXXX XXXXXX Use of the bar model 	
Column subtraction without regrouping			
Column subtraction with regrouping (Year 2+)	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters  Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones. 	Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make. 	 

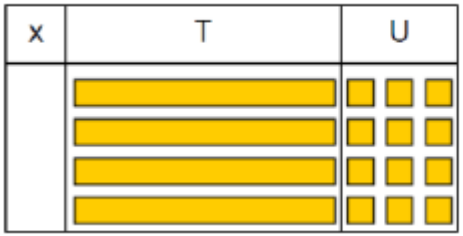
	Now I can subtract my ones.  Calculations 234 - 88 Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Calculations 234 - 88 Now I can take away eight tens and complete my subtraction  Calculations ~~2~~1~~3~~4 - 88 146 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		

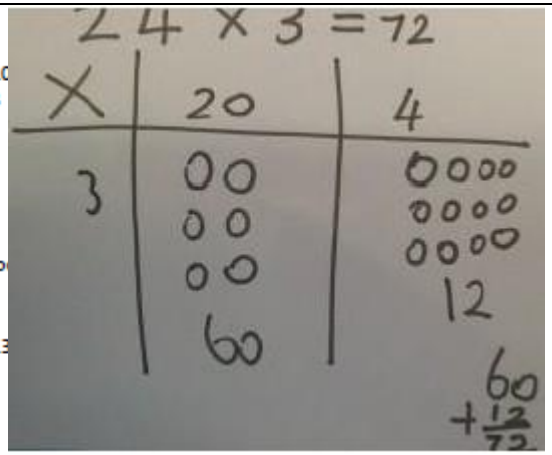
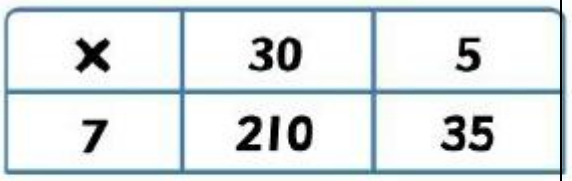
Multiplication

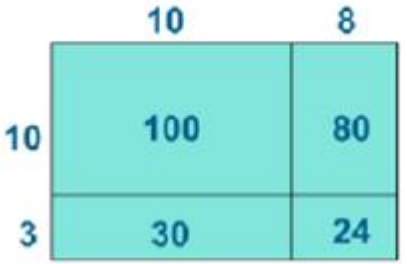
Objective	Concrete	Pictorial	Abstract
Doubling		Draw pictures to show how to double a number. Double 4 is 8 	Partition a number and then double each part before recombining it back together. 
Repeated addition	**Repeated grouping/repeated addition** (does not have to be restricted to cubes) 3 x 4 or 3 lots of 4  	Children to represent the practical resources in a picture e.g. XX XX XX XX XX XX Use of a bar model for a more structured method  Represent this pictorially alongside a number line e.g:   5 + 5 + 5 = 15 There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6	3 x 4 4 + 4 + 4 Abstract number line 3 x 4 = 12  Factor x factor = product

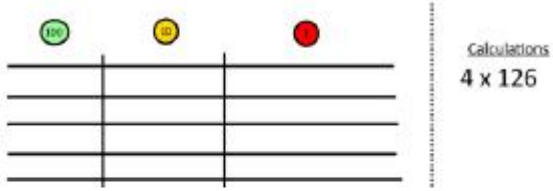
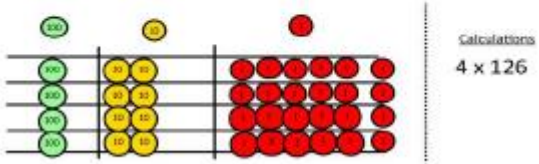
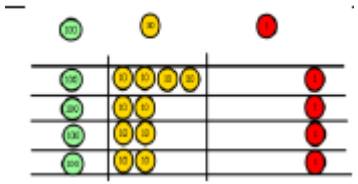
<table>
<tr>
<td>Arrays (commutative)</td>
<td> 
  </td>
<td> 

Draw arrays in different rotations to find commutative multiplication sentences.</td>
<td>5 + 5 + 5 = 15
3 + 3 + 3 + 3 + 3 = 15
5 x 3 = 15
3 x 5 = 15</td>
</tr>
<tr>
<td>Grid method (Year 3+)</td>
<td>Show the link with arrays to first introduce the grid method.
 

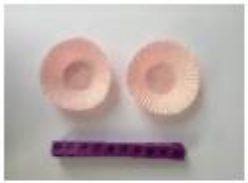
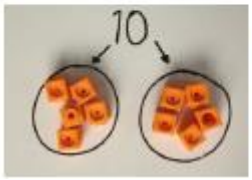
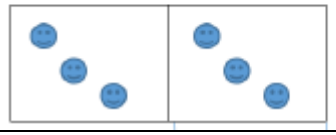
4 rows of 10
4 rows of 3
Move on to using Base 10 to move towards a more compact metho
 

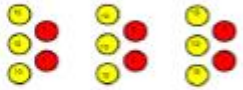
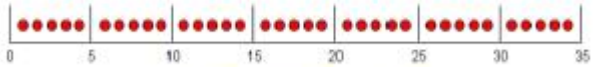
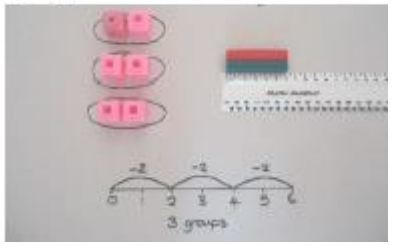
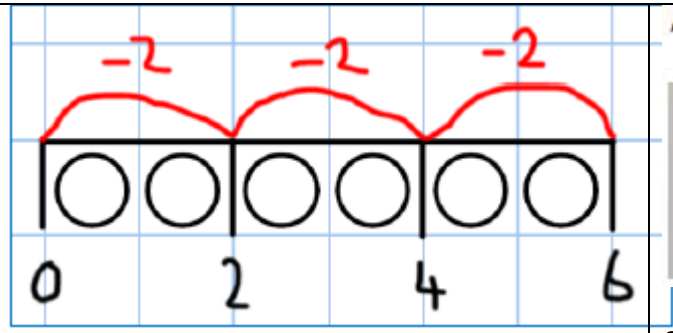
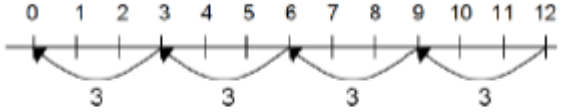
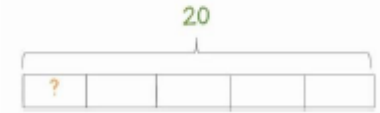
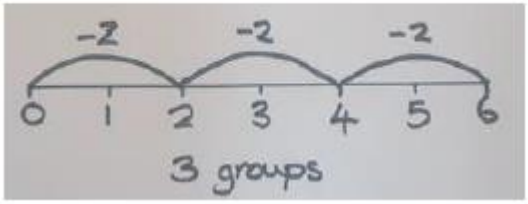
4 rows of 13</td>
<td> 
</td>
<td> 

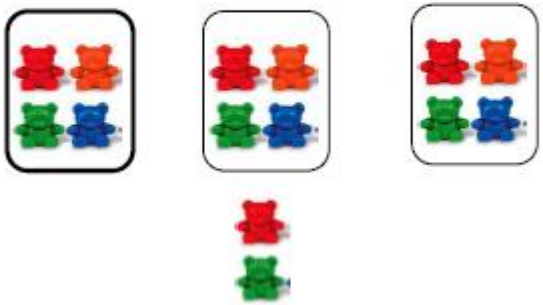
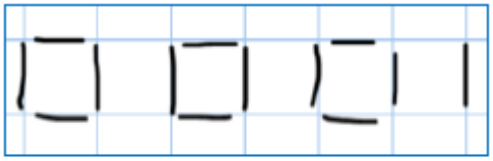
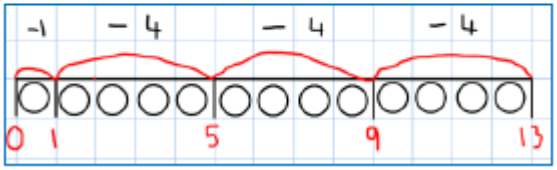
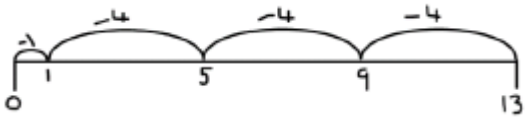
210 + 35 = 245
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.
 
</td>
</tr>
</table>

	Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.  Calculations 4 x 126 Fill each row with 126.  Calculations 4 x 126 Add up each column, starting with the ones making any exchanges needed.  Then you have your answer. 		
Column multiplication (Year 4+)	Make 23, 3 times. See how many ones, then how many tens 	**Children to represent the counters in a pictorial way**  Tens | Ones 6 | 9	Children to record what it is they are doing to show understanding 3 x 23 20 3 3 x 20 = 60 3 x 3 = 9 60 + 9= 69 23 x 3 69

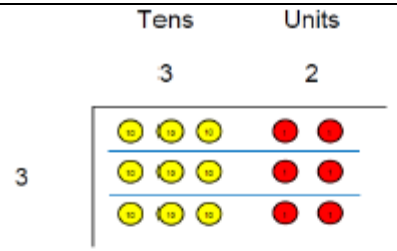
Division

Objective	Concrete	Pictorial	Abstract
Sharing objects into groups	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4  This can also be done in a bar so all 4 operations have a similar structure: 	6 ÷ 2 = 3 What's the calculation? 

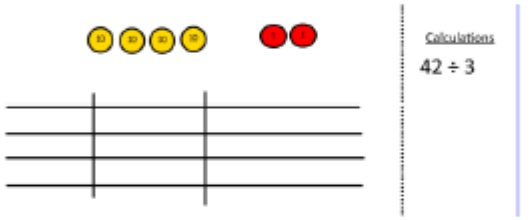
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  96 ÷ 3 = 32    6 ÷ 2  	 Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  20 ÷ 5 = ? 5 x ? = 20	Abstract number line  6 ÷ 2 = 3	

Division with a remainder (Year 2+)	**14 ÷ 3 =** Divide objects between groups and see how much is left over  Use of lollipop sticks to form wholes 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.    	29 ÷ 8 = 3 REMAINDER 5 dividend divisor quotient remainder 

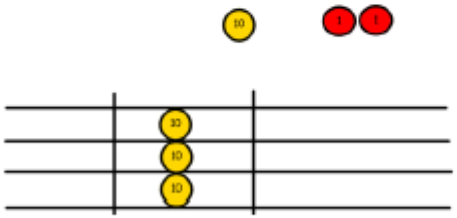
Short division (End of year 4+)



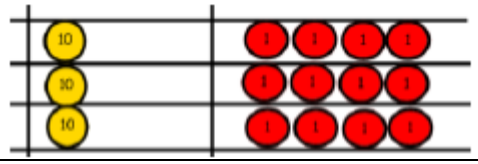
Use place value counters to divide using the bus stop method alongside



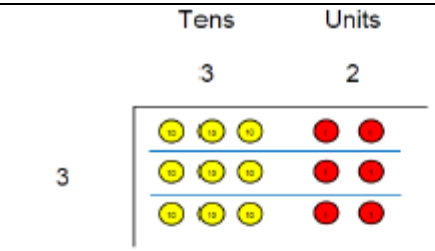
Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



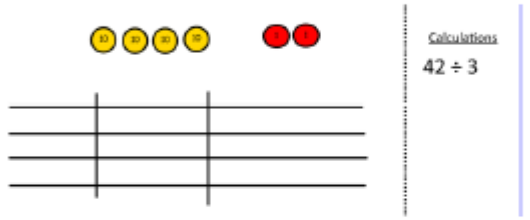
We exchange this ten for ten ones and then share the ones equally among the groups.



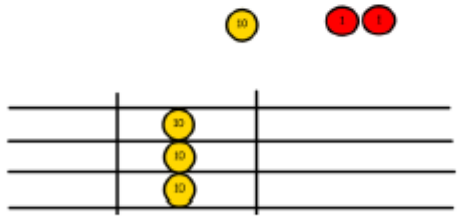
We look how much in 1 group so the answer is 14.



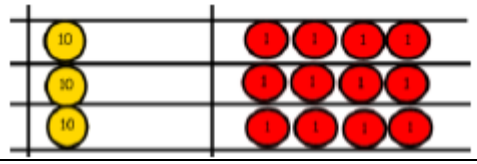
Use place value counters to divide using the bus stop method alongside



Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Begin with divisions that divide equally with no remainder.

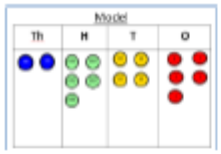
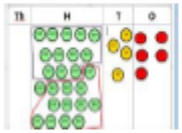
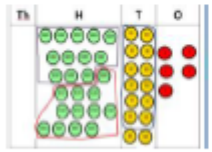
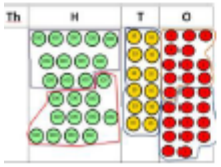
$$\begin{array}{r} 218 \\ 4\overline{)872} \\ \underline{3} \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 86\ r\ 2 \\ 5\overline{)432} \\ \underline{3} \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 35\overline{)511.0} \\ \underline{16} \\ 21 \end{array}$$

Long division (Year 5+)	 0212 12 2544 24 14 12 24 24 0 2544 ÷ 12 How many groups of 12 thousands do we have? None  Exchange 2 thousand for 20 hundreds.  02 12 2544 24 1 How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one.  021 12 2544 24 14 12 2 Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.  Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2	Children to represent the counters, pictorially and record the subtractions beneath.	0 12 2544 Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds. 02 12 2544 24 1 Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left. 021 12 2544 24 14 12 2 Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left. 0212 12 2544 24 14 12 24 24 0 Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.